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Refractivity coefficients used in the assimilation of GPS radio occultation measurements

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1 Introduction

One of the key characteristics of the GPS radio occultation (GPSRO) measurements is that they can be assimilated into operational numerical weather prediction (NWP) and reanalysis systems without bias correction. This is possible because the assimilated GPSRO quantities, usually either bending angle profiles or refractivity profiles, are derived from precise measurements of a time delay with an atomic clock. In addition, the forward problem is relatively straightforward when compared with the assimilation of satellite radiance measurements, partly because it is not considered to be reliant upon poorly known spectroscopic parameters, or the assumed concentrations of well mixed gases. In fact, the forward modelling of GPSRO measurements does require empirically derived refractivity coefficients, but to date the level of confidence in these values has been very high. The refractivity coefficients used operationally at ECMWF and the Met Office were derived in 1953 by Smith and Weintraub (SW53), but their accuracy has been reinforced in more recent work by, for example, Hasegawa and Stokesberry (1975) (HS75) and Bevis *et al.* (1994) (BEV94). In particular, there has been a broad consensus in the literature that the k_1 coefficient, which accounts for the dry air contribution to the total refractivity (see section 2), is $k_1 = 77.6 \mathrm{KhPa}^{-1}$ and this has effectively become the "standard value" used in GPSRO studies. However, Rüeger (2002) (RU02) has proposed a new set of "best average" refractivity coefficients and the new k_1 coefficient is 0.115 % larger than the standard value. The new coefficients are based on a thorough reappraisal of experimental work from the 1950's to the 1970's. RU02 also includes a small correction to account for increasing concentrations of carbon dioxide (CO_2) in the atmosphere, but this change is found to be of secondary importance here. RU02 has potentially important implications for the future development of GPSRO bending angle and refractivity observation operators, and consequently the operational level-2 temperature profiles produced with the one-dimensional variational retrieval codes.

In the context of NWP, Rüeger's coefficients were first tested for GPSRO data monitoring at Météo-France, but they were not used in assimilation experiments (Paul Poli, 2009, pers. comm.). They are now used operationally at Environment Canada (Aparicio and Deblonde, 2008). ECMWF has recently conducted forecast impact experiments comparing SW53 coefficients and the RU02 "best average" coefficients, with a view to using Rüeger's coefficients in the operational assimilation of GPSRO measurements and we have found that the RU02 coefficients cool the mean state in the troposphere by $\sim$-0.1 K. This cooling reduces the biases in the short-range forecast fit to radiosonde temperature and height measurements in the northern hemisphere, but increases the biases in the tropics and southern hemisphere. Experiments show that the cooling is caused primarily by the change in the "k_1" refractivity coefficient. Interestingly, the increased bias with respect to radiosonde measurements in the southern hemisphere appears to qualitatively similar to the results presented by Josep Aparicio (Environment Canada) at the ECMWF/GRAS SAF workshop in 2008. Aparicio uses the Rüeger coefficients operationally, but also advocates the inclusion of non-ideal gas effects in GPSRO operators in order to reduce geopotential height biases against radiosondes (Aparicio *et al.*, 2009). Furthermore, NCEP has also recently reported difficulties with the RU02 k_1 value, suggesting it is too large (Cucurull, 2009) and they have subsequently adopted the BEV94 coefficients.

Given the difficulties with the RU02 coefficients in NWP impact experiments, we have attempted to reconcile the differences between the standard (i.e. SW53/HS75/BEV94) and RU02 k_1 values. This exercise has shown that there is more uncertainty in the k_1 coefficient than is generally recognised. It has highlighted small numerical discrepancies in how the refractivity coefficients are derived from refractivity measurements by different authors, and a misunderstanding over whether the contribution of CO_2 is accounted for when deriving the coefficient. We have also investigated the connection between the choice of refractivity coefficients and the inclusion of non-ideal gas effects in the GPSRO observation operator. In section 2, we introduce and compare the standard values of the refractivity coefficients given by SW53, BEV94 and others, with the new RU02 estimates, and attempt to reconcile the difference in the k_1 coefficients. The forecast impact experiments examining the sensitivity to the SW53 and RU02 coefficients, and investigating the inclusion of non-ideal gas effects are described in section 3. The discussion and conclusions are given in section 4.

2 Refractivity Coefficients

RU02 is a detailed and comprehensive review of the radio refractivity coefficients, and the same author has also produced a summary paper: http://www.fig.net/pub/fig_2002/js28/js28_rueger.pdf. We only give the main points that are considered to be the most relevant for the GPSRO assimilation problem here, in order to convey the some of the difficulties with the published literature. We have investigated the inconsistency in the published results, but have not assessed the accuracy of individual experiments. A discussion of the refractivity coefficients is also given in Cucurull (2009).

Neglecting non-ideal gas effects, the refractivity, N, of air can be approximated with a three term expression,

$$N = \frac{k_1 P_d}{T} + \frac{k_2 e}{T} + \frac{k_3 e}{T^2} \tag{2.1}$$

where, P_d is the pressure (hPa) of dry air, e is the water vapour partial pressure (hPa), and T is the temperature (K). k_1, k_2 and k_3 are empirically derived coefficients. An alternative form of the refractivity equation is,

$$N = \frac{k_1 P}{T} + \frac{k_2' e}{T} + \frac{k_3 e}{T^2} \tag{2.2}$$

where P is the total pressure $(P = P_d + e)$ and $k_2' = k_2 - k_1$. SW53 (eq. 7) use the alternative form and give the following three term expression:

$$N = \frac{77.6P}{T} - \frac{6.0e}{T} + \frac{3.75 \times 10^5 e}{T^2} \tag{2.3}$$

but then simplify it, by combining the second and third terms by multiplying the second term by $273/T$, to give

$$N \simeq \frac{77.6P}{T} + \frac{3.73 \times 10^5 e}{T^2}. \tag{2.4}$$

This expression is currently used in operations at ECMWF and the Met Office. The $(k_1 = 77.6)$ term – which is the primary interest of this report – is derived from an average of three experiments that determined the dielectric constant, ϵ, of dry air published in 1951. SW53 quote an uncertainty $k_1 = 77.607 \pm 0.013\mathrm{KhPa}^{-1}$, which is less than 0.02 %. The SW53 k_1 has been used extensively within GPS meteorology community, but this has been justified by more recent studies. For example, both HS75 and BEV94 have performed a statistical analysis of results from 20 experiments from 1951 to 1970. HS75 suggested $k_1 = 77.600 \pm 0.032\mathrm{KhPa}^{-1}$ and BEV94 derived $k_1 = 77.60 \pm 0.05\mathrm{KhPa}^{-1}$ for their ground-based GPS studies. Furthermore, Kursinski *et al.* (1997) estimated the uncertainty in the SW53 formula for dry air to be less than 0.06 % generally, and less than 0.03 % above 500 hPa. Therefore, $k_1 = 77.6$ has essentially become the "standard value" used in GPS meteorology.

RU02 has performed a similar statistical analysis to HS75 and BEV94 and produced a new set of "best average" refractivity coefficients, given by a weighted mean of what he considers to be the most reliable experiments. After the removal of data from 11 of the experiments used by HS75 and BEV94 and the addition of one new experiment from 1977, his reanalysis of the experimental data gives

$$N = \frac{77.6848P}{T} - \frac{6.3896e}{T} + \frac{3.75463 \times 10^5 e}{T^2} \tag{2.5}$$

In addition, RU02 has adjusted the k_1 value to account for the increase in CO_2 concentration from 0.03% to 0.0375 %. This modification leads to

$$N = \frac{77.6890P}{T} - \frac{6.3938e}{T} + \frac{3.75463 \times 10^5 e}{T^2} \tag{2.6}$$

The change in k_1 as a result of the increase in CO_2 is only 0.0042 KhPa^{-1}, which is a 0.005 % increase. RU02 has estimated the uncertainty in the new, best average k_1 value is less than 0.02 % (0.015 KhPa^{-1}).

The difference between the RU02 and the standard value ($k_1 = 77.6$) of the k_1 coefficient is $\Delta k_1 = 0.089\mathrm{KhPa}^{-1}$, which is an increase of 0.115 %, more than five times the uncertainty quoted in RU02. This change in the k_1 coefficient is particularly significant for the assimilation of GPSRO data, because the use of the RU02 value produces a systematic 0.115 % increase in the forward modelled bending angles from the upper troposphere upwards (Lewis, 2008), where the measurements are given most weight in the assimilation process. The inconsistency between the k_1 value given in RU02 and the values given by HS75 and BEV94 is particularly surprising, since the authors have essentially performed the same exercise, with the aim of providing a statistically robust estimate of k_1, using the available experimental data. Two obvious reasons for the difference are apparent (see also Cucurull, 2009). Firstly, part of the discrepancy in k_1 values can be attributed to a simple numerical inconsistency, whereby it has been assumed that $0^{\circ}\mathrm{C} = 273\mathrm{K}$, when deriving k_1 from measurements of N, using $k_1 = NT/P$. This approximation has been made by a number of authors, who have tabulated experimental results, including SW53, HS75 and BEV94. As a consequence, the k_1 values derived from experiments given in Table 4 RU02 are systematically larger – by a factor of (273.15/273) or, equivalently, 0.055 % – than results from the same experiments given in both Table 1, HS75 and Table A1, BEV94. This accounts for almost half of the total difference in the k_1 values.

A second reason for the discrepancy is that the coefficients given in Table 1 of HS75 and Table A1 of BEV94 are for dry, CO_2 free air (Note that SW53 account for 0.03 % CO_2 by increasing their raw data by 0.02 %). These k_1 coefficients should be used in a four term refractivity formula, with a new term that explicitly accounts for atmospheric CO_2 contribution to the refractivity. This fact does not appear to be mentioned in BEV94, where it is used a three term expression, but it is noted in HS75 (p 871). The RU02 k_1 value includes 0.0375 % of CO_2. Including 0.0375 % CO_2 increases a CO_2 free k_1 value by $\sim$ 0.02. If we combine this modification with the error associated with mis-specifying $0^{\circ}\mathrm{C}$, we can increase the HS75/BEV94 k_1 value from $k_1 = 77.60$ to $k_1 \simeq 77.66$, which is two thirds of the $\Delta k_1 = 0.089$ value.

The remaining differences between the k_1 values are probably attributable to data selection. RU02 only uses data from nine of the twenty experiments included by HS75 and BEV94. For example, RU02 disputes the use of refractivity data which has been extrapolated from the visible or infrared, which tends to bias the derived k_1 values low. In summary, the k_1 coefficient suggested in RU02 appears to be more robust and defendable than the standard value.

Including Non-ideal gas effects when deriving k_1 from N

An additional complication is the inclusion of non-ideal gas effects when deriving k_1. In general, most authors appear to derive the k_1 from a measured N using $k_1 = NT/P$, neglecting non-ideal gas effects by assuming the density is given by $\rho = P/RT$, but in some cases these details are unclear. For example, SW53 state in connection with the data given in their Table 1, "These values are also given on a real rather than an ideal gas basis.", but they then use the ideal gas equation when deriving their k_1. Similarly, the data given in Table 1 of Boudouris (1963) is "after the Van der Vaals correction for real gases for normal temperature and pressure". However, further details of the nature and size of the correction are not given.

Thayer (1974) introduced a more general, three term expression for the refractivity,

$$N = \frac{k_1 P_d}{Z_d T} + \frac{k_2 e}{Z_w T} + \frac{k_3 e}{Z_w T^2} \tag{2.7}$$

where P_d is the pressure of dry air and Z_d and Z_w are the dry air and water-vapour compressibilities, respectively. The compressibilities account for non-ideal gas effects, such as the finite size of the molecules and mutual attraction, and they are a function of pressure, temperature and water vapour pressure. In the atmospheric conditions of interest, the compressibility is less than unity, so their inclusion in eq. 2.7 increases the computed value of N. One interesting implication of eq. 2.7 is that if refractivity coefficients are derived neglecting non-ideal gas effects, they are not really constant because each coefficient has incorporated the compressibility factor, which is itself a function of the atmospheric state.

Thayer (1974) has included non-ideal gas effects when deriving k_1 from a measurement of N. In fact, he derives (eq. 6, Thayer, 1974) an estimate of k_1 from the same experimental data and refractivity value used by SW53, $N = 288.04 \pm 0.05$, but correctly uses $0^{\circ}\mathrm{C} = 273.15\mathrm{K}$. The coefficient is given by $k_1 = (NTZ_d)/P$, where $(1/Z_d = 1.000588)$ is the inverse of the compressibility of dry air for P=1013.25 hPa and T=273.15 K. The computed k_1 value is

$$k_1 = 288.04 \times \left(\frac{273.15}{1013.25 \times 1.000588} \right) = 77.604 \tag{2.8}$$

It is interesting to note that the SW53 error associated with assuming $0^{\circ}\mathrm{C} = 273\mathrm{K}$ is almost completely cancelled as a result of the additional error caused by ignoring non-ideal effects; (273.15/273)= 1.000549 and $1/Z_d = 1.000588$, when P=1013.25 hPa and T=273.15 K. This means that the Thayer and SW53 k_1 values are in very good agreement. Unfortunately these values should differ because they are used in different formulations of the refractivity equation. Kursinski *et al.* (section 3.9.1, 1997) based their error estimate for the SW53 k_1 coefficient of less than "0.03 % above mbar", on comparisons with Thayer's expression. However, note that RU02 disputes Thayer's k_1 value because, like SW53, it is partly based on an optical measurement which has been extrapolated to radio wavelengths, and this is known to bias the k_1 estimate low. Cucurull (2009) has tested the Thayer refractivity coefficients in a refractivity equation which does not include compressibility. This approach is questionable because the Thayer coefficients have been reduced to account for their use in an equation that includes compressibility (e.g., note the 1.000588 factor in eq. 6, Thayer, 1974).

RU02 does not include non-ideal gas effects when deriving k_1 from experimental N values, and it is conceded that the introduction of compressibility requires further evaluation. Introducing a compressibility factor of $Z_d = 1/1.000588$ would reduce the RU02 best average value from $k_1 = 77.689$ to $k_1 = 77.643$. RU02 argues that non-ideal gas effects should only be included when deriving k_1, if the k_1 value is going to be used in an expression for N like eq. 2.7 that includes the compressibility terms. Conversely, if k_1 is derived ignoring non-ideal effects, it should only be used in an expression for N that does not include compressibility (e.g., eq. 2.2). It appears that either approach is equally valid, provided that we are consistent in both the derivation and use of the coefficients, since by construction both give the same refractivity value at P=1013.25 hPa and T=273.15 K, but in general these approaches give slightly different results because the compressibility factor is a function of pressure and temperature.

3 Forecast impact experiments

There is clearly a degree of uncertainty in the refractivity coefficients that arises as a result of assumptions made by different authors when processing the experimental refractivity data. This has probably not been fully appreciated within the GPSRO community, so it is important to establish whether it has any significant implications for the assimilation of GPSRO data in operational NWP and reanalysis applications. Therefore, we have investigated the analysis and forecast sensitivity to the refractivity coefficients used in the assimilation of bending angle profiles in a series forecast impact experiments. Similar work has been presented recently by Cucurull (2009), who investigated the sensitivity of refractivity profile assimilation to the assumed coefficients in the NCEP assimilation system. The experiments presented here use the CY35R1 version of the ECMWF analysis system and are run in the incremental four-dimensional variational assimilation (4D-Var) configuration, at T159/159 resolution, with 91 levels in the vertical. The experiments cover the period December 1, 2008 to January 31, 2009 and assimilate all the data types that were used operationally during this period. Variational bias correction (VarBC, Dee 2005) is used to correct the satellite radiance measurements. The GPSRO measurements from the COSMIC constellation and the GRAS instrument on MetOP-A are assimilated with a one-dimensional (1D) bending angle operator (Healy and Thépaut, 2006). The control experiment (CTL) uses the current operational implementation of the 1D bending operator, with the SW53 refractivity coefficients (eq. 2.4). The Rüeger experiment (RUEG) is identical to the CTL experiment, except for the use of RU02 best available coefficients in a three term expression (eq. 2.6), to evaluate the refractivity on the model levels in the 1D bending angle operator.

Figure 3.1 shows COSMIC-4, noise normalised observation minus background (o-b) bending angle departures, in the southern hemisphere for the RUEG and CTL experiments. As expected (Lewis, 2008), the bending angles simulated with the Rüeger coefficients are larger in the upper troposphere and stratosphere, so the (o-b) departures are systematically smaller. This negative shift in the departures is evident for all instruments and all geographic regions. Figure 3.2a shows the zonally averaged mean temperature analysis differences (RUEG minus CTL) averaged from December 1, 2008 to January 31, 2009. The use of the Rüeger coefficients produces a systematic cooling below 300 hPa. This cooling partially compensates for the larger forward modelled bending angles, by reducing the height of the model levels. The 300 hPa geopotential differences are shown in Figure 3.2b, giving an indication of the spatial variation of the cooling. The mean geopotential height of pressure levels above 300 hPa is $\sim$ 5 m lower in the RUEG experiment, south of 50 degrees south. We have verified that these changes are primarily caused by the k_1 coefficient by running a "mixed" experiment, using the RU02 k_1 value in SW53 two term formula (eq. 2.4) giving,

$$N = \frac{77.689P}{T} + \frac{3.73 \times 10^5 e}{T^2}. \tag{3.1}$$

The results using this expression for refractivity are almost identical to the RUEG experiment in terms of the temperature and height biases, confirming that the difference in the k_1 coefficient is the most significant change.

The tropospheric cooling produces mixed results when the short-range forecasts are compared with radiosonde temperature and height measurements (Figures 3.3 and 3.4). In the northern hemisphere the temperature biases are improved from 850 hPa to 150 hPa by typically a few hundredth's of

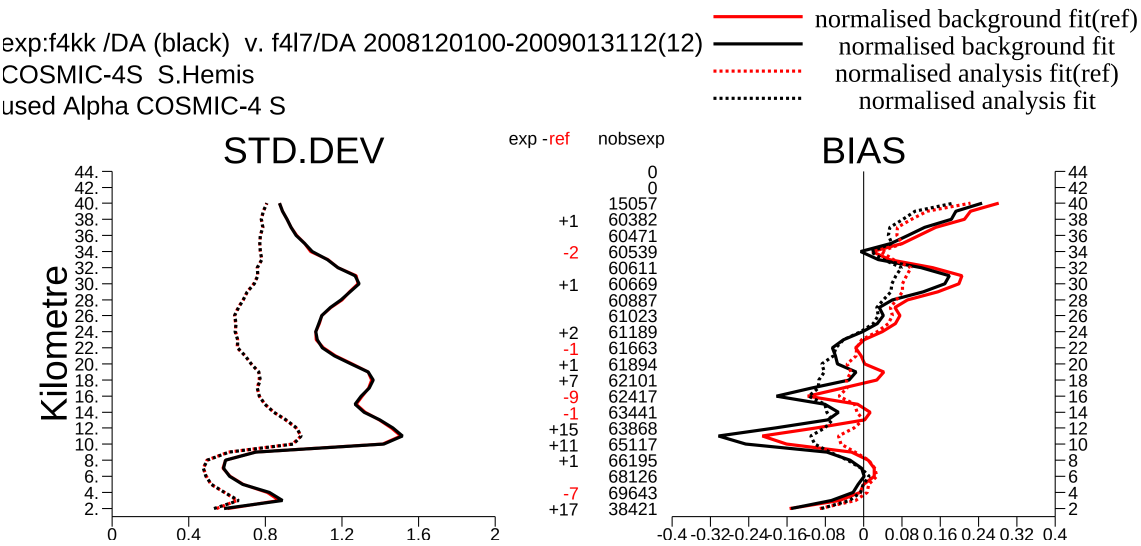

Figure 3.1: The standard deviation and mean of the noise normalised bending angle departures for the RUEG(black) and CTL(red) experiments in the southern hemisphere averaged over the period December 1, 2008 to January 31, 2009. The (o-b) departures are solid lines and the (o-a) departures are dotted lines.

Kelvin, and the geopotential height departures are generally better except near 250-300 hPa. The negative temperature bias against radiosondes in the northern hemisphere is probably related to the assimilation aircraft temperature measurements which are biased positive, and tend to produce negative temperature bias against radiosondes. This problem is most significant in the northern hemisphere where the number of aircraft temperature measurements is greatest. The results in the tropics and southern hemisphere show that the RU02 coefficients tend to increase the positive temperature biases below 500 hPa and geopotential height biases throughout the atmosphere. In the southern hemisphere the bias at 100 hPa increases from 6.1 m to 9.3 m and in the tropics it increases from 8.7 m to 11.4 m. Note that the results shown in Figures 3.3 and 3.4) are for all radiosonde types, but restricting comparisons to just RS92 radiosonde measurements does not change the main results.

In general, the forecast scores are neutral when the RUEG and CTL experiments are verified against their own analysis. The forecast scores against temperature and geopotential height observations in the tropics and southern hemisphere are slightly worse for the RUEG experiment, primarily as a result of increased temperature biases in the troposphere.

Including Non-ideal gas effects in a GPSRO operator

Non-ideal gas effects are normally neglected in NWP, but at the ECMWF/GRAS-SAF workshop Josep Aparicio suggested that they needed to be included in the GPSRO observation operators to reduce a forward model bias (see also Aparicio *et al.*, 2009). Aparicio has implemented the RU02 refractivity coefficients and found that the assimilation of GPSRO refractivity profiles increases the bias of short-range forecasts with respect to radiosonde height measurements. Aparicio's results are qualitatively similar to those given section 3, but he found that he could reduce the bias by including non-ideal gas effects in his refractivity observation operator. Non-ideal gas effects have now been introduced into the 1D bending angle operator and this has been tested in the assimilation system.

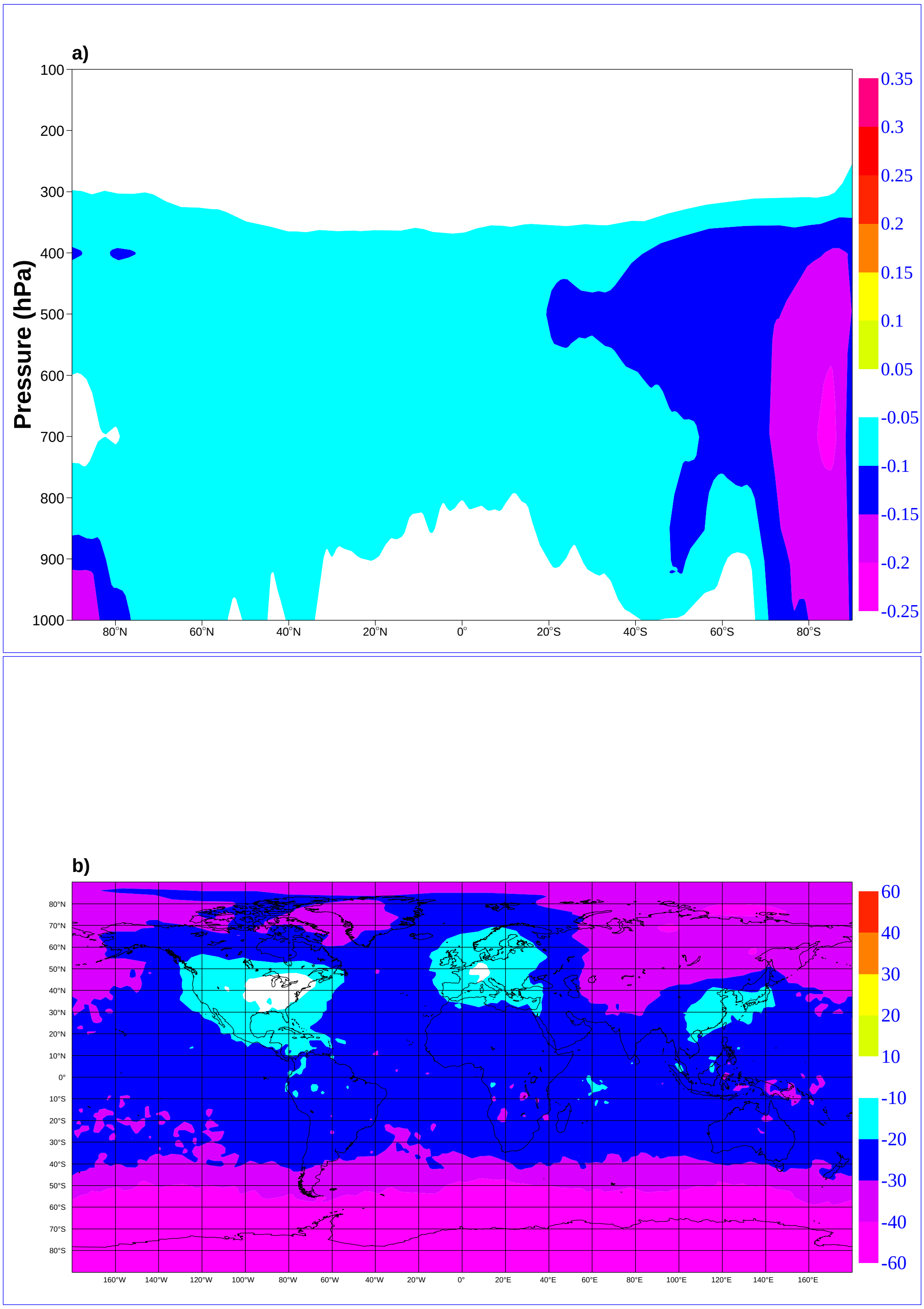

Figure 3.2: a) shows the zonally averaged temperature analysis differences (RUEG-CTL) averaged over the period December 1, 2008 to January 31, 2009. b) shows the corresponding geopotential differences for the 300 hPa surface.

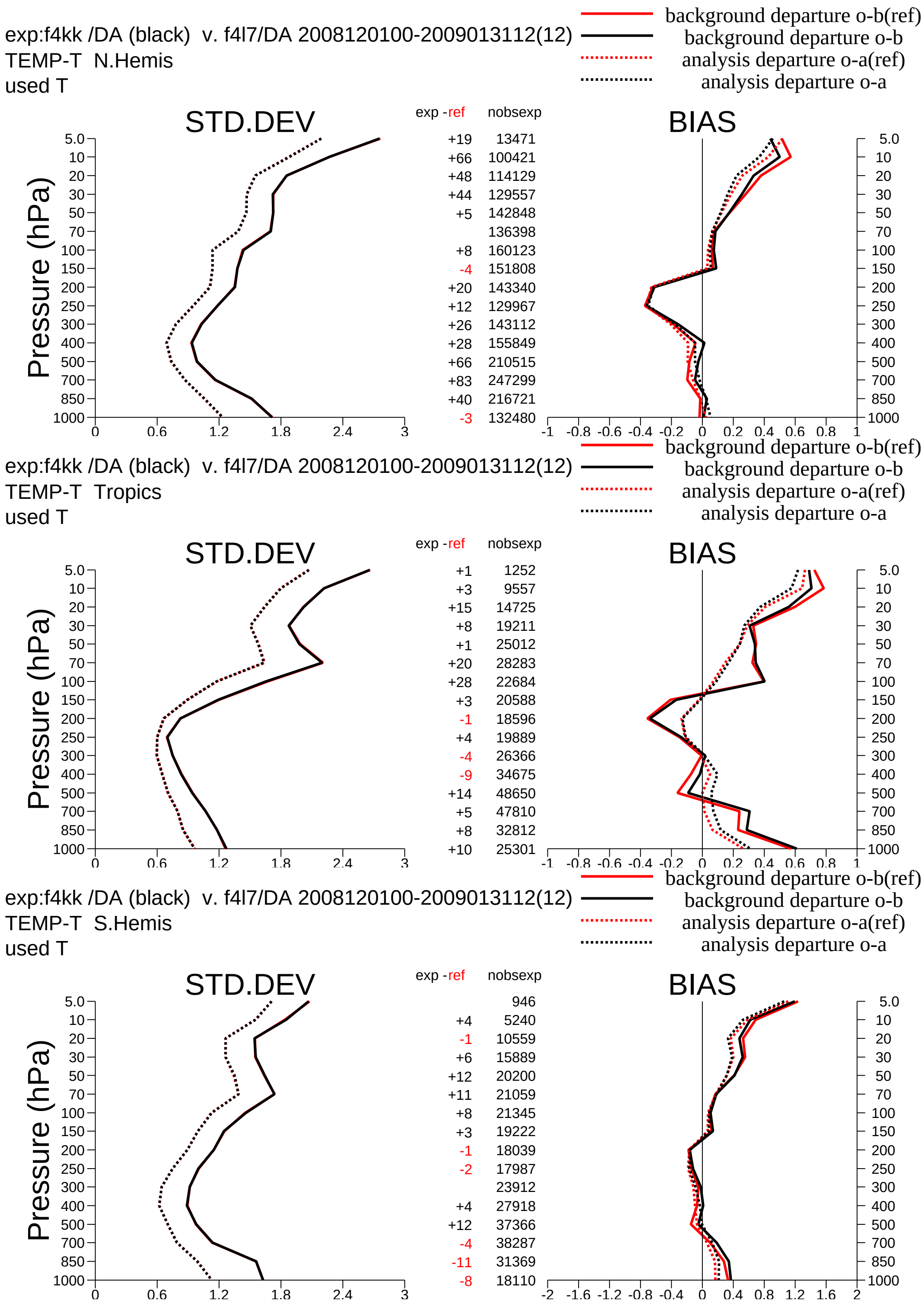

Figure 3.3: The standard deviation and mean of the radiosonde temperature departures in the northern hemisphere (top), tropics (middle) and southern hemisphere (bottom). The RUEG experiment is the black line and the CTL is the red line.

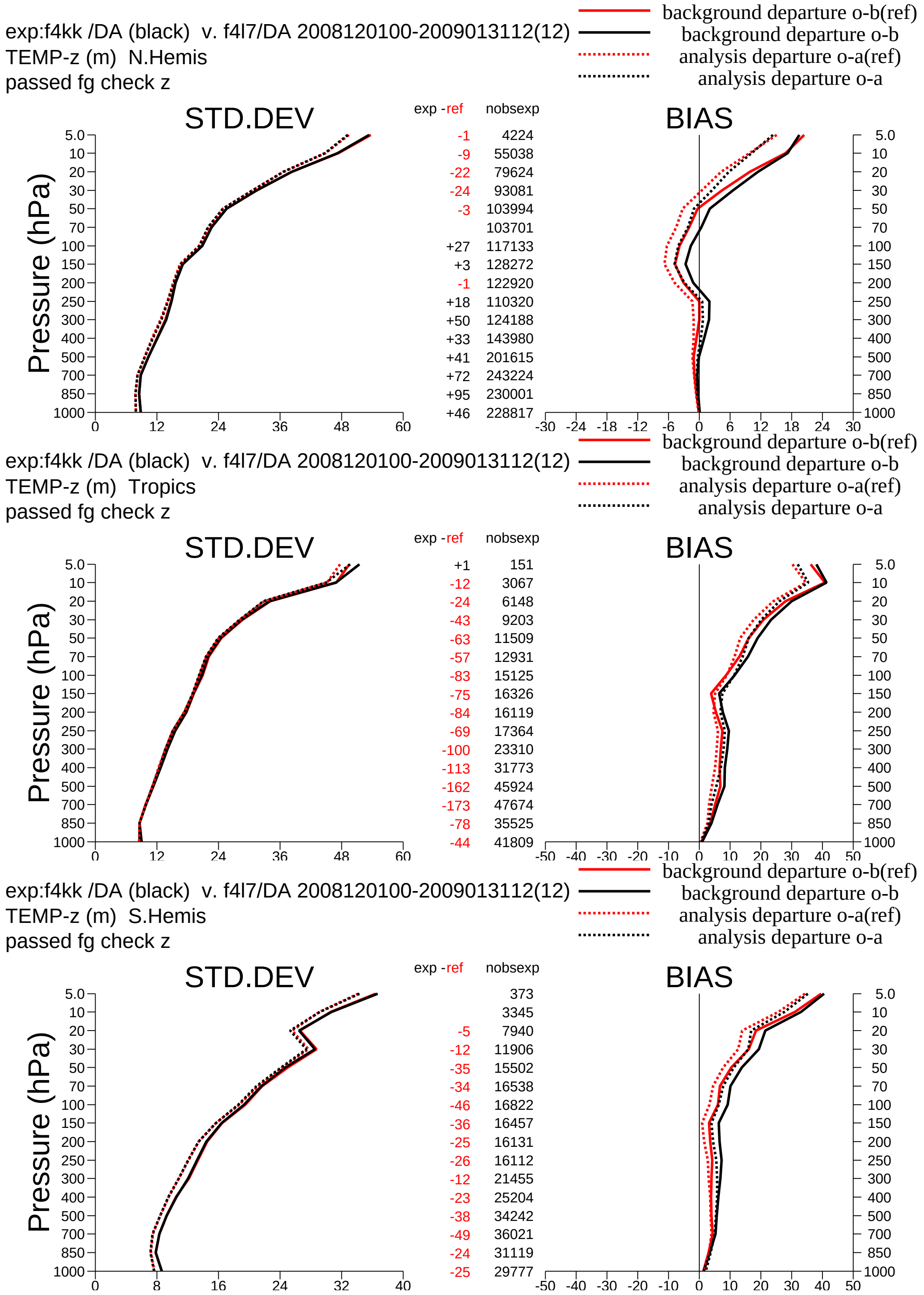

Figure 3.4: The standard deviation and mean of the radiosonde geopotential height departures in the northern hemisphere (top), tropics (middle) and southern hemisphere (bottom). The RUEG experiment is the black line and the CTL is the red line.

The bending angle observation operator contains essentially three major components. The first is the integration of the hydrostatic equation, to compute the height of the model levels. The second is the evaluation of refractivity on the model levels and the third is the evaluation of the bending angle integral. Non-ideal gas effects should be included in both the integration of the hydrostatic equation and the evaluation of the refractivity on the model levels. Note that Aparicio *et al.* (2009) only consider the impact on the hydrostatic integration.

Following Aparicio *et al.* (2009) the non-ideal gas equation of state can be written as,

$$P = \rho R T_v Z \tag{3.2}$$

where P is the total pressure, ρ is the density, R is the gas constant for dry air, T_v is the virtual temperature and Z is the compressibility of moist air. For an ideal gas $Z = 1$, but for air in the troposphere typically $Z \sim 0.9995$, so the departure is 0.05 %. Davis (1992) and Picard *et al.* (2008) provide a polynomial expansion for Z, which is straightforward to implement in observation operators. If we include the compressibility in the hydrostatic integral, the geopotential height, h, becomes

$$h = -\int \left(\frac{Z R T_v}{g_o P} \right) dp \tag{3.3}$$

where $g_o = 9.80665\mathrm{ms}^{-2}$. Given that $Z < 1$, it is clear that including compressibility reduces the height of model levels. In the bending angle operator, the thickness between the ith and $(i+1)th$ model levels calculated neglecting non-ideal gas effects (Simmons and Burridge, 1981), $\Delta h'_{i,i+1}$, is scaled by the average of the compressibility at ith and $(i+1)th$ levels, to give $\Delta h_{i,i+1} = 0.5 \times (Z_i + Z_{i+1}) \times \Delta h'_{i,i+1}$. In simulations, we have found that height of model levels near 100 hPa can be reduced by $dh \sim 7\mathrm{m}$. Assuming a refractivity scale height of 7 km, $\Delta N/N = -dh/7\mathrm{km}$, this translates into a reduction in the forward modelled refractivity or bending angle value of $\Delta\alpha/\alpha = -0.1\%$. The second step where non-ideal effects should be included is the computation of refractivity on the model levels using (Thayer, 1974)

$$N = \frac{k_1 P_d}{Z_d T} + \frac{k_2 e}{Z_w T} + \frac{k_3 e}{Z_w T^2} \tag{3.4}$$

where Z_d and Z_w are the compressibility of the dry air and water vapour, respectively. Note how the combined effects of these changes partially cancel. The heights of the model levels are reduced, meaning that the refractivity on a given height level is smaller. However, the inclusion of compressibility in the refractivity computation increases the value. In fact, if the compressibility factor was a constant for the entire profile, it would have little impact on the simulated bending angle values, because the hydrostatic and refractivity computation changes should cancel each other out.

The impact of introducing non-ideal gas effects has been investigated in three experiments. In the first experiment, “COMP1”, the compressibility has been introduced in the hydrostatic integration, but not in the computation of refractivity on the model levels (which uses the Rüeger coefficients). This experiment essentially reproduces the approach adopted by Aparicio *et al.* (2009), but applies it to a bending angle operator rather than a refractivity operator. In the second experiment, “COMP2”, compressibility is used in both the hydrostatic integration and refractivity computation, where Rüeger’s coefficients (eq. 2.6) are used in eq. 3.4. The third experiment, “COMP3”, is identical to “COMP2” except that Rüeger’s “k_1” value has been adjusted downwards to $k_1 = 77.643 = (77.689/1.000588)$, to account for the fact it is being used in a refractivity formula that includes compressibility factors. Similar adjustments have not been made to the k_2 and k_3 values, but it has already been noted that these terms do not have a major impact on the temperature and geopotential height biases shown here.

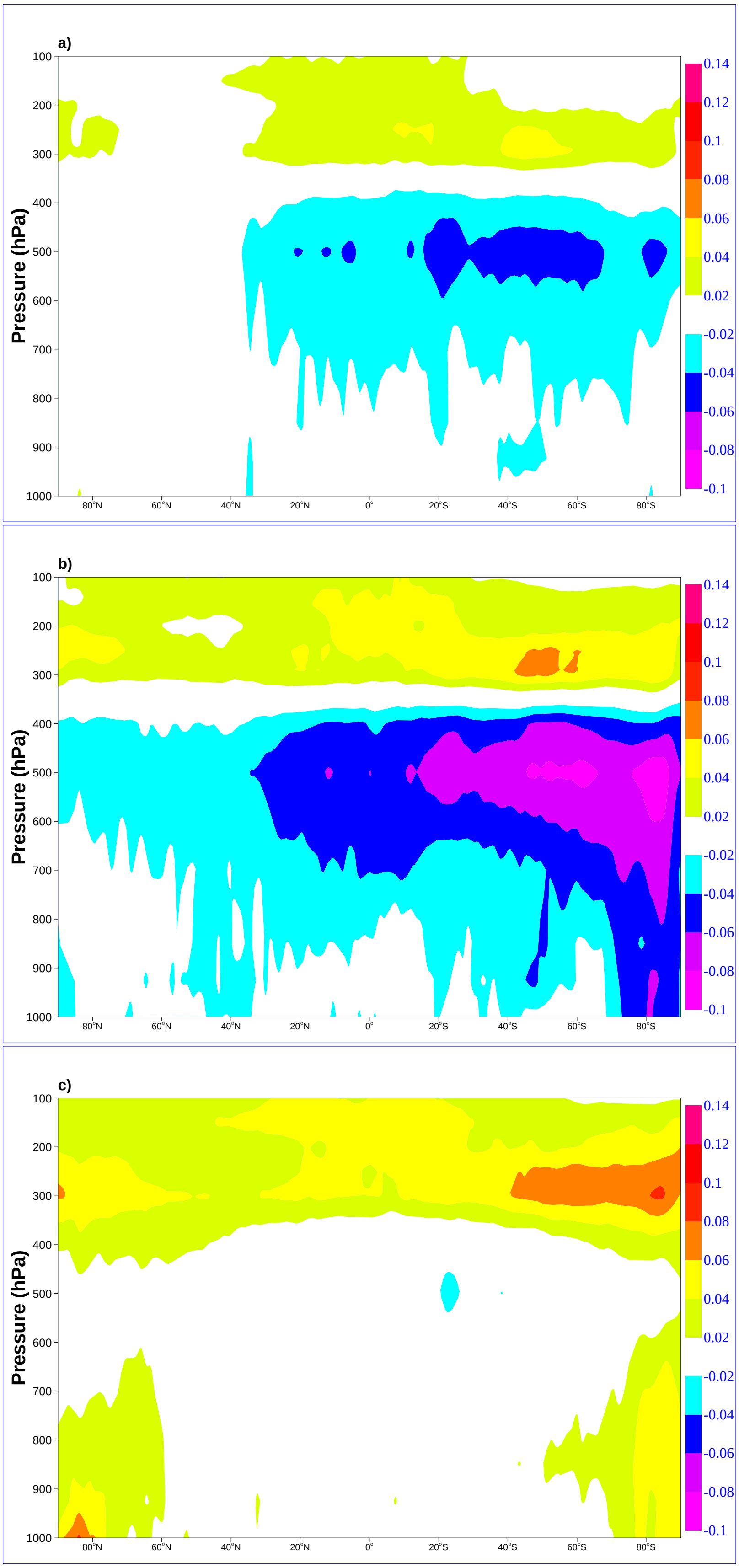

Figure 3.5: The zonal average temperature analysis differences a) COMP1-CTL, b) COMP2-CTL and c) COMP3-CTL.

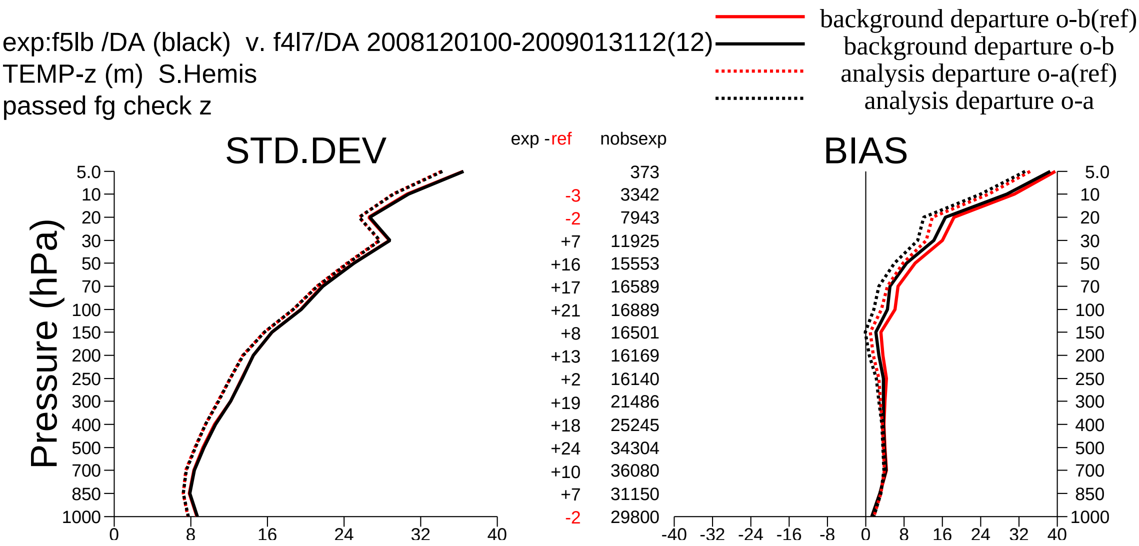

Figure 3.6: The standard deviation and mean of the radiosonde geopotential height departures in the southern hemisphere. The COMP3 experiment is the black line and the CTL is the red line.

The inclusion of compressibility reduces the simulated bending angle values by $\sim 0.1\%$ in the lower/mid stratosphere and improves the agreement with the CTL experiment. The mean temperature analysis differences of the COMP1, COMP2 and COMP3 minus the CTL experiments are shown the in Figure 3.5. These should be compared with Figure 3.2, noting that the temperature range has reduced in Figure 3.5. It is clear that the inclusion of compressibility has in general improved the agreement with the CTL experiment, but the level of improvement varies for each experiment. The COMP1 experiment performs better than COMP2, in terms of reducing the temperature biases relative to the CTL. This is because in the COMP2 experiment the reduction of the simulated bending angles that is a result of including compressibility in the height calculation is partly cancelled by the increase in the refractivity values that is a result of including compressibility in the expression for refractivity. However, although COMP1 produces slightly better results, it is a partial approach, which is difficult to justify on theoretical grounds. The difficulty with COMP2 is because the RU02 coefficients are being used in an expression that includes compressibility, when they were derived assuming this was not the case. This inconsistency has been removed in COMP3, where the k_1 value has been reduced. The implementation tested in COMP3 appears to be the most theoretically consistent, and in fact the COMP3 experiment has a better fit to the radiosonde height measurements in the southern hemisphere than the CTL experiment (Figure 3.6). These results clearly demonstrate that the introduction of compressibility can have a significant impact on the fit to radiosonde measurements.

Note that GRAS GPSRO bending angle departures are currently biased negative in operations at ECMWF and COSMIC departures are expected to shift negative when testing of the new smoothing procedure of the phase delays is completed (Sergey Sokolovskiy, 2009, pers comm). The implementation of COMP3 could reduce some of this bias, but the magnitude of the bias reduction will depend on which value of the k_1 coefficient is used in the forward model.

4 Discussion and conclusions

The results presented above have demonstrated the analysis and forecast sensitivity to the k_1 refractivity coefficient used to forward model the bending angle observations. It has been shown that an increase in the k_1 of 0.115 % suggested in RU02 leads to a systematic increase in the simulated bending angle values in the stratosphere. This results in a cooling in the troposphere which improves radiosonde biases in the northern hemisphere but degrades the fit to radiosonde temperature and height measurements in the tropics and southern hemisphere. Nevertheless, the evidence in support of the RU02 values appears to be strong. Indeed his detailed analysis has highlighted some problems with the interpretation of the estimates given by HS75 and BEV94, namely:

1) assuming $0^{o}\mathrm{C} = 273\mathrm{K}$ when deriving k_1 from a measured N.

2) the coefficients are valid for dry, carbon-dioxide free air.

It is clear that the HS75 and BEV94 studies should not be cited in support of the SW53 value. Despite these deficiencies, Cucurull (2009) has recently adopted the BEV94 values for operational use at NCEP, based on the pragmatic approach that they produce the better forecast scores. This illustrates that performance within an NWP system is not a reliable method for assessing the most accurate coefficients, because the NWP model itself will be biased.

One possible limitation with the RU02 best average values are that they neglect non-ideal gas effects, and it is noted that this simplification requires further evaluation. Neglecting non-ideal gas effects when deriving the refractivity coefficients produces coefficients that are weakly dependent on the atmospheric state. Furthermore, in the context of GPSRO assimilation, compressibility should also be included in the integration of the hydrostatic equation; Aparicio *et al.* (2009) have recently demonstrated that including non-ideal gas effects in a GPSRO refractivity forward model that uses the RU02 coefficients improves the short-range forecast biases with respect to radiosonde geopotential height measurements in the southern hemisphere. We have essentially reproduced these results with a bending angle forward model, and shown that the RU02 coefficients with non-ideal gas effects, gives similar results to the control experiment, using SW53 and neglecting non-ideal gas effects. We obtain the best results in southern hemisphere when compressibility is included in both the hydrostatic integration and the evaluation of refractivity, and with the RU02 value reduced to $k_1 = 77.643$, to account for its use in an expression that includes compressibility. Therefore, it appears in some instances we may have previously obtained the right results for the wrong reasons when using the SW53 values operationally. However, this should not detract from the fundamental problem, which is the uncertainty in the k_1 refractivity coefficient. This study reinforces the need for new measurements because it is not possible to determine the best coefficients from the assimilation results. On the contrary, new, accurate measurements of the coefficients should be used in the GPSRO forward models, and then we need to investigate how the assimilation system reacts to them. Nearly all of the experimental measurements date from the 1950's and 1960's, and RU02 considers the best available measurements for deriving k_1 were performed by Newell and Baird (1965). BEV94 acknowledged this problem and stated that " we would like to see some contemporary determinations of the refractivity coefficients", and Cucurull (2009) has recently expressed a similar view. In addition, some clarification of how and when non-ideal gas effects are included is required, because this is unclear in some of the older literature.

All GPSRO forward modelling and retrieval techniques are reliant on accurate knowledge of the refractivity coefficients, and it appears that previous estimates of their accuracy have been too optimistic. It is now clear that the systematic error in the forward modelled bending angle values may be of order ~ 0.1 %, as a result of the uncertainty in the refractivity coefficients . As noted earlier, one of the strengths of GPSRO measurements is that they can be assimilated without bias correction, and as a consequence they anchor the bias correction of satellite radiances in operational NWP and reanalysis (ERA-Interim) systems. In addition, it has also been suggested that GPSRO measurements be used in climate monitoring and climate model testing activities. (Leroy *et al.*, 2006). Clearly, uncertainty in the refractivity coefficients will also have some impact on these applications.

In conclusion, we have examined the sensitivity to the assumed refractivity coefficients and confirmed the results of Aparicio *et al.* (2009), showing including non-ideal gas effects in GPSRO bending angle operator have a demonstrable impact on NWP analyses. Modern measurements of the refractivity coefficients at radio frequencies would have important applications in operational NWP, reanalysis and climate activities and they should be encouraged.

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