Formation-Flying CubeSat Constellations for Internal Gravity Wave Tomography

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Outline

• Formation Flying for Clustered Occultations
• Occultation Cluster Quality and Distribution
• Constellation Maintenance and Lifetime
• Conclusions
Formation Flight & Sounding Clusters
Previous Work: RO Tomography of Gravity Waves

- Technique demonstrated with COSMIC-1 data, before satellites dispersed: Wang and Alexander, JGR (2010); Schmidt, Alexander, de la Torre (2016). Sensitive only to long wavelengths (~1000 km) and low frequencies (~1/day) in order to find any clusters.

- A dedicated mission can find short wavelength (~50 km), high frequency waves (~1/30 min). Otherwise ~10,000 randomly distributed LEO satellites would be needed to develop an RO momentum flux climatology of these waves.

Alexander et al., QJRMS (2010)
Mutual Orbit Groups

- Satellites slightly perturbed in inclination, RAAN, and eccentricity can be made to “orbit” each other, and orbit the Earth as a group.
- “Mutual Orbit Groups” can be constructed in any orbit, with any number of satellites.
- (The orbit angular momentum vectors and eccentricity vectors should be distributed in cones.)
Cluster Distribution and Quality
Distribution of Occultation Clusters

- MOGs can yield clustered soundings spaced evenly around the world (within latitude constraints).
- This would not be consistently achieved by random or string-of-pearls constellations.
Occultation Cluster Quality Metrics

- $\chi^2$ minimization relates sounding position to uncertainty in wave vector reconstruction.

$\chi^2 = \frac{1}{\sigma^2} \sum_{i=1}^{n} (k \cdot r_i + \phi_0 - \phi_i)^2$

- $q_1 \propto$ uncertainty area
- $q_2 \propto$ least-certain axis

$$R = \begin{bmatrix} \delta x_1 & \delta y_1 \\ \vdots & \vdots \\ \delta x_n & \delta y_n \end{bmatrix}$$

$$q_1 = \sigma_\phi^2 \left( \det R^T R \right)^{-1/2}$$

$$q_2 = \sigma_\phi \left[ \lambda_{\min}(R^T R) \right]^{-1/2}$$
Quality Metrics \((\sigma_\phi = 1)\) v. Latitude
Constellation Maintenance
Out-of-Plane Thrusts

- Symmetric arcs of thrust at the maximum and minimum latitudes can counter $J_2$ torques.
- Angle $\gamma$ gives required thrust arc length:
  - Impulsive propulsion lets $\gamma$ approach zero.
  - Electric propulsion requires that $\gamma$ be a substantial fraction of the orbit arc.
Burn Arc Length and Efficiency

- Rate of $\Delta V$ use is determined by ideal rate ($\alpha^*$, from orbit geometry) and efficiency ($\eta$, from burn arc):

$$\bar{\alpha}_j \equiv \frac{\alpha^*_j}{\eta} \quad \eta = \text{sinc} \frac{\gamma}{2}$$

- For correction every $N^{th}$ orbit with thrust/mass $\alpha_{max}$:

$$\bar{\alpha}_j = \frac{2\alpha_{max}}{N\pi} \sin^{-1} \left( \frac{N\pi}{2} \frac{\bar{\alpha}^*_j}{\alpha_{max}} \right)$$
Rate of $\Delta V$ Expenditure

- Example (ISS):
  - $i = 51.4^\circ$
  - $\delta = 0.172^\circ$
  - $h = 400 \text{ km}$

- Frequent burns or higher thrust per mass raises efficiency.
Maintenance & SmallSat Propulsion

• Trade study is necessary to choose propulsion option:
• High-thrust options (e.g. cold gas) able to correct for larger spacecraft separations. They have small $\gamma$ and high $\eta$.
• Low-thrust options (e.g. electrospray) have appreciably higher total $\Delta V$ capacity, and therefore longer lifetime despite less efficient operation.
Constellation Maintenance Lifetime

- Simulations performed for 10 kg small satellite, varying propulsion option.
- Rate of $\Delta V$ expenditure is driven by inclination separation of satellites.
- Increased specific impulse of electric propulsion provides greatly increased lifetime if correction is possible.

Further correction impossible ($\gamma = \pi$)
Conclusions

• Formation-flying satellites can be used to yield frequent occultation clusters for tomography of internal gravity waves.
• Long burn arcs with electric propulsion necessary for long lifetimes, but mutual orbit group separation is limited by available thrust/mass of available systems.
• Ability to perform precision orbit determination during low-thrust burn arcs drives occultation yield of the constellation if long lifetimes are desired.
Questions?

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